CALCULUS.

261. Proposed by S. A. COREY, Hiteman, Iowa.

Prove that $\sum_{x=1}^{x=\infty}\frac{1}{a+2bx^2+cx^4}=\frac{\pi}{\sqrt{[8ac(\sqrt{ac}+b)]}}-\frac{1}{2a}$, where $ac>b^2$.

Solution by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

$$\sum_{x=1}^{x=\infty}\frac{1}{a+2bx^2+cx^4}=\int_1^\infty\frac{dx}{a+2bx^2+cx^4}=A.$$

As $ac>b^2$, $a+2bx^2+cx^4=(\sqrt{a}+2kx+x^2\sqrt{c})(\sqrt{a}-2kx+x^2\sqrt{c})$, where $k=\sqrt{\frac{\sqrt{(ac)}-b}{2}}$.

$$\therefore A=\frac{1}{4k\sqrt{a}}\int_1^\infty\frac{(x+2k)dx}{\sqrt{a}+2kx+x^2\sqrt{c}}-\frac{1}{4k\sqrt{a}}\int_1^\infty\frac{(x-2k)dx}{\sqrt{a}-2kx+x^2\sqrt{c}}$$

$$=\left[\frac{1}{8k\sqrt{a}}\log\frac{\sqrt{a}+2kx+x^2\sqrt{c}}{\sqrt{a}-2kx+x^2\sqrt{c}}+\frac{1}{4\sqrt{[a(b+k^2)]}}\tan^{-1}\frac{2x\sqrt{(b+k^2)}}{\sqrt{a}-x^2\sqrt{c}}\right]_1^\infty$$

$$=\tfrac{1}{2}\left[\frac{\pi}{\sqrt{ah}}-\frac{1}{a}\right],\text{ where } h=2\sqrt{(ac)}+b, =\frac{\pi}{\sqrt{[8ac(\sqrt{ac}+b)]}}-\frac{1}{2a}.$$

Also solved by G. B. M. Zerr.

262. Proposed by H. SCHAFFER, Fayetteville, Ark.

Prove that the circle is the only plane curve of constant curvature.

Solution by C. N. SCHMALL, New York City.

The expression for the curvature of a plane curve, $F(x, y)=0$, is

$$\frac{1}{R}=\frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}=c,\text{ say}\ldots(1).$$

Put $\frac{dy}{dx}=z$. $\because\frac{d^2y}{dx^2}=\frac{dz}{dx}$, and (1) becomes $\frac{dz/dx}{(1+z^2)^{\frac{3}{2}}}=c$, whence

$dx=\frac{dz}{c(1+z^2)^{\frac{3}{2}}}$, and, therefore, $x=\frac{1}{c}\int\frac{dz}{(1+z^2)^{\frac{3}{2}}}=\frac{1}{c}\cdot\frac{z}{\sqrt{(1+z^2)}}$.